The Game of Craps

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1 How to Play Craps

Craps is a game played with a pair of dice. In the game of craps, the shooter (the player with the dice) rolls a pair of dice and the number of spots showing on the two upward faces are added up. If the opening roll (called the ‘coming out roll’) is a 7 or 11 (called ‘throwing a natural’), the shooter wins the game. If the opening roll results in a 2 (‘snake eyes’), 3 or 12 (‘box cars’), the shooter loses, otherwise known as ‘crapping out’. If the shooter rolls a 4, 5, 6, 8, 9 or 10 on the opening roll, then he or she must roll the same number (called ‘making the point’) before rolling a 7 to win the game. For example, if the shooter rolls a 6 on the come out roll, a 10 on the second roll and a 7 on the third roll, the shooter loses since he or she rolled a 7 before rolling another 6. If, however, if the shooter rolled a 6 on the third roll, he or she wins the game.

2 What is the Probability of Winning Craps?

Let $B$ denote the event of winning at craps. In addition to $B$, we define the events below.

$A_1$: the shooter throws a 7 or 11 on the first roll (a natural)

$A_i$: the shooter throws an $i$ on the opening roll,

$i = 4, 5, 6, 8, 9, 10$, and eventually makes his/her point

Clearly, since the events are mutually exclusive,


Observe that $P(A_1) = P(throwing a 7 or an 11) = P(throwing a 7) + P(throwing an 11) = 6/36 + 2/36 = 8/36$. Now consider the probability of
making the point. Specifically, consider computing $P(A_4)$. Let $C$ denote the event of shooting something other than a 4 or a 7. Observe that $P(C) = 27/36$. Using independence, we can compute $P(A_4)$ as follows. Since

$$A_4 = (4 \text{ on first roll } \cap 4 \text{ on second roll}) \cup (4 \text{ on first roll } \cap C \cap 4 \text{ on third roll}) \cup (4 \text{ on first roll } \cap C \cap C \cap 4 \text{ on fourth roll}) \cup \cdots$$

we have that

$$P(A_4) = P(4 \text{ on first roll } \cap 4 \text{ on second roll}) + P(4 \text{ on first roll } \cap C \cap 4 \text{ on third roll}) + P(4 \text{ on first roll } \cap C \cap C \cap 4 \text{ on fourth roll}) + \cdots$$

$$= \left( \frac{3}{36} \right) \left( \frac{3}{36} \right) + \left( \frac{3}{36} \right) \left( \frac{27}{36} \right) \left( \frac{3}{36} \right) + \left( \frac{3}{36} \right) \left( \frac{27}{36} \right) \left( \frac{27}{36} \right) \left( \frac{3}{36} \right) + \cdots$$

$$= \left( \frac{3}{36} \right)^2 \sum_{k=0}^{\infty} \left( \frac{27}{36} \right)^k$$

$$= \left( \frac{3}{36} \right)^2 \left( \frac{36}{9} \right)$$

$$= \frac{1}{36}$$

We can compute $P(A_5)$, $P(A_6)$, $P(A_8)$, $P(A_9)$, and $P(A_{10})$ is a similar manner. The following table summarizes all the probabilities.

<table>
<thead>
<tr>
<th>Event</th>
<th>$P(\text{Event})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$8/36$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$\frac{1}{36}$</td>
</tr>
<tr>
<td>$A_5$</td>
<td>$\frac{25}{36}$</td>
</tr>
<tr>
<td>$A_6$</td>
<td>$\frac{25}{36}$</td>
</tr>
<tr>
<td>$A_8$</td>
<td>$\frac{25}{36}$</td>
</tr>
<tr>
<td>$A_9$</td>
<td>$\frac{10}{36}$</td>
</tr>
<tr>
<td>$A_{10}$</td>
<td>$\frac{1}{36}$</td>
</tr>
</tbody>
</table>
Therefore, the probability of winning at craps is


Craps is one of the only casino games that comes close to being a *fair* game. A game is fair if the probability of winning is .50.