Instructions. There are no special instructions for these practice problems.

1. On the material since Test #2, you should know...

   - Chapter 5 (excluding Section 4), Section 6.2 and Section 6.4
   - what the three characteristics of decrease-and-conquer algorithms are
   - how to describe a decrease-and-conquer algorithm for exponentiation (by one)
   - how to describe a decrease-and-conquer algorithm for exponentiation (by a factor of two)
   - how to describe a decrease-and-conquer algorithm for the gcd (variable decrease)
   - how to perform a BFS of an undirected or directed graph and compute the arrays $T[v], D[v], \text{ and } P[v]$
   - what the worst-cost of BFS is
   - what types of edges are generated in a BFS of a undirected or directed graph (tree, back and cross)
   - how to use BFS to determine if an undirected graph is bipartite
   - how to use BFS to determine if an undirected graph has a cycle
   - how to use BFS to identify the connected components of an undirected graph
   - how to perform a DFS of an undirected or directed graph and compute the arrays $D[v], F[v], \text{ and } P[v]$
   - what types of edges are generated in a DFS of a undirected or directed graph (tree, back, forward and cross)
   - what the worst-cost of DFS is
   - how to use DFS to determine if an undirected graph is bipartite
   - how to use DFS to determine if an undirected graph has a cycle
   - how to use DFS to identify the connected components of an undirected graph
   - how to use DFS to identify the strongly connected components of a directed graph
   - how to define the transitive closure matrix of a directed graph
   - how to use DFS to compute the transitive closure matrix of a directed graph
   - what a DAG is
   - how to define a topological sort of a DAG
   - how to perform a topological sort of a DAG
   - how to solve the Fake Coin Problem
   - how to do multiplication à la Russe
- how to compute the Josephus number
- what the Selection Problem is
- what the $k^{th}$ order statistic is
- a linear-time (average case) algorithm for the Selection Problem
- what the three variations of transform-and-conquer are
- how to define a full binary tree having $n$ nodes and depth $d$
- how to define a complete binary tree having $n$ nodes and depth $d$
- how to define a heap
- a linear-time algorithm for heap construction (bottom up)
- HeapSort
- the array implementation of HeapSort
- what a $n \times n$ linear system is
- what the three row operations are
- how to perform Gaussian elimination to obtain the solution to an $n \times n$ linear system
- what the efficiency class of Gaussian elimination is

2. A certain Loyola student loves foreign languages and wants to plan her course schedule to take the following nine language courses: ML15, ML16, ML22, ML31, ML32, ML126, ML127, ML141, and ML169. The prerequisites are

- ML15 (none)
- ML16 : ML15
- ML22 : (none)
- ML31 : ML15
- ML126 : ML22, ML32
- ML127 : ML16
- ML141 : ML22, ML16
- ML169 : ML32

Find a sequence of courses that allows the students to satisfy all the requirements.

3. Let $G$ be a graph whose vertices are the integers 1 through 8, and let the adjacent vertices of each vertex be given by the table below.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Adjacent Vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,3,4</td>
</tr>
<tr>
<td>2</td>
<td>1,3,4</td>
</tr>
<tr>
<td>3</td>
<td>1,2,4</td>
</tr>
<tr>
<td>4</td>
<td>1,2,3,6</td>
</tr>
<tr>
<td>5</td>
<td>6,7,8</td>
</tr>
<tr>
<td>6</td>
<td>4,5,7</td>
</tr>
<tr>
<td>7</td>
<td>5,6,8</td>
</tr>
<tr>
<td>8</td>
<td>5,7</td>
</tr>
</tbody>
</table>

(a) Draw $G$
(b) Do a BFS of $G$ and output the arrays $T[v]$, $D[v]$, and $P[v]$ (use numerical order to break ties)

(c) Do a DFS of $G$ and output the arrays $D[v]$, $F[v]$, and $P[v]$ (use numerical order to break ties)

4. Explain why there are no forward edges with respect to the BFS tree constructed for a directed graph.

5. Let $G$ be a directed graph whose vertices are the integers 1 through 7. The edges are as follows: (6,1), (2,3), (4,2), (7,2), (3,4), (4,3), (6,4), (7,4), (6,7), (5,6), and (5,7). Note that $(i, j)$ means that vertex $i$ is directed toward vertex $j$.

   (a) Draw $G$.
   (b) Write down the transitive closure matrix of $G$.
   (c) Write down the strongly connected components of $G$.

6. Multiply 490 and 2468 by multiplication à la Russe.

7. Find $J(3287)$.

8. Find the median element of the following array using Algorithm Select that was described in class. Display the matrix after each application of Split.

   \[ \begin{array}{cccccccc}
   16 & 33 & 56 & 11 & 50 & 31 & 7 & 49 \\
   \end{array} \]

9. Consider the array in the preceding problem.

   (a) Transform the array into a heap (keeping the data in the array). Display the array after each siftdown operation.
   (b) Sort the array using HeapSort (keeping the data in the array). Display the array after each swap and siftdown operation.

10. p. 207, #1